Abstract

The purpose of this study was to compare students’ performance in a freshman level quantitative reasoning course (QR) under three different course sequence models. A cohort of 155 freshman students was placed in one of the three models: needing a prerequisite course, corequisite (students enroll simultaneously in QR course and a course that provides remediation) or ready to take the QR course alone. A chi-square test revealed a statistically significant relationship between the students’ final course grades and course models. Furthermore, an ANOVA test indicated that students’ grades in the QR course were higher when they completed the course under the corequisite model compared to the prerequisite model. Our study concludes that students’ performance and perceptions were significantly higher when they completed the QR course under the corequisite model compared to the prerequisite model.

Introduction

Regis College is a private liberal arts institution that enrolls approximately 1,000 undergraduates annually. In an effort to improve the quantitative literacy skills of our students and to advance STEM education, the college has made significant curriculum changes recently. As part of this initiative, a quantitative reasoning (QR) course has been introduced in the freshman year in lieu of traditional college algebra courses. In essence, QR is a college-level mathematics course that focuses on critical thinking and problem solving skills using real life applications. Topics covered in this course include logic, arguments, reasoning and problem solving, mathematical finance (loan, credit card, mortgage), tax, federal budget, linear, and exponential models as well as some geometry related topics. In order to compete in a global economy that is driven by data and overloaded with information, QR skills are essential to our young people. In light of these emerging changes, organizations like AAC&U (The Association of American Colleges and Universities), which focuses on improving undergraduate education and advancing liberal education, has recognized the importance of quantitative or mathematical skills in the population at large.

Quantitative literacy is one of the LEAP (Liberal Education for America’s Promise) Essential Learning Outcomes of a number of practical intellectual skills, including inquiry and analysis, critical and creative thinking, written and oral communication, information literacy and teamwork, and problem solving (Elrod, 2014). Similarly, the Mathematical Association of America (MAA) highlighted in a report that all colleges and universities should treat quantitative literacy as a thoroughly legitimate and even necessary goal for baccalaureate graduates (Quantitative Reasoning for College Graduates: A Complement to the Standards, Summary, 1994).

In general, QR has been defined in different ways. According to Elrod (2014): quantitative reasoning is the application of basic mathematical skills, such as algebra, to the analysis and interpretation of real-world quantitative information in the context of a discipline or an interdisciplinary problem to draw conclusions that are relevant to students in their daily lives (Quantitative Reasoning: The Next “Across the Curriculum” Movement, para.7). QR utilizes basic mathematics skills in the service of carrying out complex reasoning and decision-making processes. It is less about how to present the calculation and more about the meaning of the calculation results (Steen, 2004). Additionally, many researchers such as Moore, Carlson, & Oehrtman (2009) point out that QR is an essential skill for problem solving and is critical to sustained success in mathematics. Other researchers including Smith & Thompson (2008) argued that by focusing on QR, students will be able to conceptualize, improve reasoning, and operate on quantities in a sensible manner while solving problems. They also suggest that QR will help students to bridge the gap between algebraic and arithmetic reasoning as it relates mathematics to the real world applications.

The quantitative reasoning is an essential skill for students in science, mathematics and statistics to model real world phenomenon related to biology, environment, energy and other fields. It is an indispensable tool in the practice of STEM and enhances students’ engagement in STEM fields, as suggested in a recent paper by Rocconi and colleagues (2013) that reported students in STEM fields are more engaged in QR-related activities than those in non-STEM fields. We recognized the importance of QR in various STEM fields and its interconnection across different fields of study. The goal was to create a curriculum that addresses the QR needs of all of our students by providing meaningful engagement in mathematics that will simultaneously develop quantitative literacy and spark an interest in various STEM fields. In order for Regis College to have a well-defined STEM program, through which a student may become quantitatively literate, the college decided to design and implement a comprehensive quantitative literacy program based on the guidelines established by the Mathematical Association of America. These guidelines state: Colleges and universities should devise and establish quantitative literacy programs each consisting of a foundational experience and a continued experience. Mathematics departments should provide leadership in the development of such programs (Quantitative Reasoning for College Graduates: A Complement to the Standards, Summary, 1994). At Regis, QR fulfills the foundational experience and after the successful completion of this course students continue to pursue other mathematics courses such as statistics or calculus along with other courses in quantitative literacy linked disciplines.

During the 2014–2015 academic year, about 300 students entered Regis College largely consisting of biology, business, communications, social sciences, nursing and health science professions and a handful of mathematics education majors. The average mathematics SAT scores for these students was 484 (the middle 50% had a range of 440-535) with an average high school GPA of 3.01. We placed 155 freshman students into three different course sequence models of QR courses as detailed below. The goal was to determine which of the following models was most effective in terms of students’ performance and satisfaction. In fact, the motivation for our study stems from the need for alternate remedial approaches for freshmen students to enhance their mathematics learning and quantitative literacy skills. Our placement procedure used ACCUPLACER (ACCUPLACER, 2015) as a tool to identify student aptitude and quantitative skills. We also considered other factors such as SAT score, GPA, and mathematics courses that were previously taken in high school before making the final placement decisions. We allowed
freshmen students to repeat the ACCUPLACER test if they felt they did not perform as they would have expected. In short, our placement system considered multiple tools and data from various systems for placement to maximize reliability and validity of the placement process. 

**QR alone:** Students scoring between 70–100 on ACCUPLACER with SAT score between 450–500 and high school GPA of at least 2.75, were enrolled in the 3-credit QR course without any need for remediation. We randomly placed 85 freshman students who required remediation in two different sequence models of QR course based on their ACCUPLACER score along with SAT score and high school GPA, and it was determined by the mathematics department.

**Prerequisite model:** These students were enrolled in a 1-credit remedial course to improve their arithmetic and elementary algebraic skills before proceeding to the 3-credit QR course during the subsequent semester.

**Corequisite model:** These students took both the QR course and the required remedial course simultaneously. In this model, students earned 1-credit for the remedial work and 3 credits for the QR course.

As mentioned earlier, our goal was to assess the effectiveness of the corequisite model in the newly introduced QR course. This model is becoming an increasingly accepted approach to college level mathematics remediation to accelerate student progress and readiness for advanced college level mathematics courses (Cullinane, 2012). Early evaluations of this model suggest that the corequisite approach is associated with higher grades and higher completion rate in introductory college level courses in less time and with significant savings for students and institutions (Transform Remediation: The Co-Requisite Course Model, 2012). It also improves persistence and higher total credit accumulation for students (Jenkins et al, 2010).

**Method of Study**

The data for this study was collected from 155 students who entered Regis College in the academic year 2014–2015 as freshmen. Out of these students, 46 of them were placed in the prerequisite model, 39 of them were placed in the corequisite model and 70 of them were placed directly in QR alone. There were 6 sections of this course that were taught by three different experienced instructors (tenured/tenure track faculty) but all used a common syllabus and grading criteria. In order to maintain the homogeneous nature of this group, these instructors used identical instructional methods and resources for QR courses and met regularly to discuss the progress and concerns that they had in these classes. The tests, quizzes and homework assignments were also identical in content. The textbook used for this course was “Using and Understanding Mathematics: A Quantitative Reasoning Approach” by Bennet and Briggs (2014). Students completed their homework assignments using the Pearson online homework module, MyMathLab (Pearson Education, 2014). However, the tests and quizzes were completed in class proctored by the instructors throughout the semester. The final course grade for each student was calculated at the end of the semester. This was calculated using a weighted average formula comprising quizzes (12 percent), three semester tests (48 percent), online homework (16 percent), attendance and class participation (4 percent), and the final exam (20 percent). The data collection was finished in fall 2014 for those who completed the QR course under corequisite and QR alone models. However, for the students in the prerequisite model, we had to wait until the end of spring 2015 to have the complete data. These students completed the prerequisite in fall 2014 and then proceeded to the QR course in spring 2015.

**Design of Corequisite Model**

The students in the corequisite sequence model had to meet an additional 90 minutes every week with the instructor. In the beginning of these sessions, diagnostic tests were administered to students to assess their level of proficiency in basic mathematical concepts and operations. Later on, supplemental instructions were provided to students based on this initial assessment. For these sessions, we selected the instructional materials that would strengthen their understanding and skills in mathematics related to the content to be covered in the QR course. This corequisite instruction involved college-level QR course topics along with integrated review of developmental algebra. The instruction included worksheets and assignments from MyMathLab, which were designed to help students master these topics. The prerequisite instruction included the same topics but was not directly linked to QR course topics as implemented in the corequisite model.

At this point, we would like to discuss in detail how we integrated the corequisite model in the QR course with four sample problems and the linked student learning outcomes (SLOs) that they fulfill. The following problems, or similar ones, were used in tests and quizzes in order to assess students' ability to achieve the listed learning outcomes.

**Problem 1 & 2 (SLO: Demonstrate the ability to model and represent exponential growth and decay. SLO: ability to represent quantitative information using graphs)**

**Problem 1:** Consider an antibiotic that has a half-life in the bloodstream of 12 hours. A 10-milligram injection of the antibiotic is given at 7:00 p.m. How much antibiotic remains in the blood at 8:00 a.m. when you are ready to take the medication the following morning? Draw a graph that shows the amount of antibiotic remaining as the drug is eliminated by the body.

Solving this problem demands students to have the skills in dealing with exponential notation and their operations, as they would be using the formula:

\[ \text{New Value} = \text{Initial Value} \times \left( \frac{1}{2} \right)^{\text{Time}} \]

To make sure students are comfortable with the mathematical operations involving exponentials in QR class, the instructor conducted a review on exponential notations and their calculations during the supplemental session. The instructor solved several problems involving this topic and then students were asked to work in different groups on many similar exercises in class. This was to ensure that students gained the necessary skills and confidence in dealing with these calculations during the supplemental sessions before proceeding to solve the above problem.

Here we present some sample problems practiced during the supplemental session: compute \( 4^{12} \) using a calculator and also check your answer by manual calculations using the properties of exponents. Thereafter, students were guided to solving equations like \( 5^{x+2} = 100 \). These exercises led the students to understand the relation between logarithms and exponential functions as well as how to solve them by switching between these two different formats. A series of problems on this topic were assigned as homework as well. Our goal was to use the supplemental sessions to build these skills before solving the Problem 1 in the QR class.

We observed that with supplemental instructions and practices, most students became quite confident in dealing with logarithms and exponents. We conducted post-tests on these topics and students' performance was analyzed during these sessions. The students were referred to the Quantitative Center (Q-Center) if they continued to experience difficulty with understanding the content or mastering the skills. The Regis College Q-Center serves the undergraduate and graduate student populations by providing comprehensive, drop-in tutoring support for courses with quantitative components. The faculty member teaching the course would gather the data from the Q-Center to confirm if students were on track and to identify any weak areas both in instruction and student learning. Once the instructor determined that the students had achieved the required level competency, they were asked to solve Problem 1 and similar problems of increasing difficulty.

**Problem 2:** At 11:00 you place a single bacterium in a bottle, and at 11:01 it divides into 2 bacteria, which at 11:02 divide into 4 bacteria, and so on. The bottle will be full at 12:00.

(a) How many bacteria will be in the bottle at 11:56?
(b) What fraction of bottle will be filled at 11:56?
(c) At what time will the bottle be half-full?
(d) At what time will the bottle be 10% full?
Students were also asked to compare examples of exponential change and, based on the experience gained from examining these problems, students explained the connection between logarithms and exponents. We found that the supplemental session offered under the corequisite model and the simultaneous application of those concepts linked to the actual QR course, made a huge impact in students’ motivation and problem solving skills as reflected in the student survey listed in Table 5.

Problem 3 (SLO: Explore and interpret rates of change, contrasting linear versus exponential growth (simple versus compound interest))

Problem 3: The Wall Street Journal reports that students graduating in 2011 (year 0) will have an average debt of $22,900 and it is increasing to $24,732 one year later, we might suppose that the debt increases by $1,832 each year or we might think it increases by 8% each year.

(a) Fill in each of the tables below. In the left table assume that each year the debt increases by $1,832 per year. On the right assume that each year the debt is 8% larger than the previous year.

(b) Create the formula of a linear function to model the average college debt in the left table.

(c) Based on the model from the preceding question, what will the average college debt be in 2045 when the present freshmen’s children may be graduating?

(d) Create an exponential function to model the average college debt in the right table.

(e) Based on the model from the preceding question, what will the average college debt be in 2045 when the present freshmen’s children may be graduating?

Problem 4 (SLO: Operate within and between different measurement scales including unit conversion and dimensional analysis).

Problem 4: You are planning to make pesto and need to buy basil. At the grocery store, you can buy small containers of basil priced at $2.99 for each 2/3-ounce container. At the farmer’s market, you can buy basil in bunches for $12 per pound. Which is the better deal?

We want to emphasize here that QR assessment has been an ongoing process at many institutions. The James Madison University provides a comprehensive assessment tool in QR (Sundre, Thelk & Wigtil, 2008) for this purpose. In our pilot study we used the assessment of SLOs in different course sequence models along with other items such as quizzes, tests, homework, and the final exam along with final course grades. These results are discussed in the following section along with student perceptions gathered through a survey.

Data and Results

We gathered both qualitative and quantitative data from students to assess their performance and experiences under these models. We analyzed our data for all students taking QR and also for students in each of the models using the statistical software SPSS.

The results of this study are given in Tables 1-5: To analyze results effectively, we created several categories of data: Students in QR alone model, corequisite model, prerequisite model, and all students. For each of these categories, the sample size, mean and standard deviation of the final course grade are displayed in Table 1.

Table 2 displays the contingency table and chi-square test result for the three categories of students that constitute our entire sample. We counted the number of students who earned C- or above (passing grade) and the number of students who received D+ or below (failing grade) in each of these three categories (observed count). We performed a chi-square test to find out whether this association between the course models and the grade earned (pass/fail) is statistically significant. The results of the chi-square test given in Table 2 suggest that these variables are dependent.
Null Hypothesis stated that grades obtained (≤ D+ and ≥ C-) and course models are independent (i.e., no relationship between the two variables). Alternate hypothesis stated that grades obtained (≤ D+ and C- ≥) and course models are dependent.

Test results: chi-square test statistic $\chi^2 = 9.03$, df = 2, $0.01 < p-value < 0.025$.

Because $p-value < 0.05$, we rejected the null hypothesis in favor of the alternate hypothesis. Based on this test, we concluded that two variables (course models and pass/fail grades) are dependent at alpha = 0.05 level of significance.

The chi-square test only revealed some association between the course model and grades (pass/fail). Therefore, to determine the difference in course grades among three different models, we performed an ANOVA test by comparing the means of final course grades under these models.

Null hypothesis stated that all three means are same and alternate hypothesis stated that at least two of them are different.

Our test results: F test statistic = 6.504, p-value = 0.002

Because $p-value < 0.05$, we rejected the null hypothesis in favor of the alternate hypothesis. Based on this test, we concluded that at least two of the three means are different. Because we had three pairs of means to compare, we performed a multiple-comparison ANOVA test. We found that the mean of the final course grade for two pairs (prerequisite model, corequisite model, p-value = 0.004) and (prerequisite model, QR alone, p-value = 0.001) were statistically different. We did not observe any difference between the means of the final course grade for corequisite model and QR alone.

We used an assessment rubric that reported scores for each of the SLOs mentioned above. Using this information we calculated the percentage of students who achieved a total score of 60% on problems 1-4 or similar problems. The mathematics department, in consultation with the college wide assessment committee, accepted this benchmark of 60%. This assessment data (Table 4) show a similar pattern as in Table 1. In fact, these results support the overall grades that students gained in different course models. Currently, we are designing a QR Placement Test to replace ACCUPLACER that students will be required to take before enrolling in a QR course and a sampling of questions from the Placement Test will appear again on the final exam that will enable us to assess the gains made over the semester.

**Discussion**

Our analysis (Table 1 and 3) shows that students in the corequisite model had a significantly higher average course grade compared to students in the prerequisite model and again these students have received a higher percentage of B's and C's. Similarly, Table 4 shows that a higher percentage of students in the corequisite model have achieved the accepted benchmark for SLOs compared to the prerequisite model.

Overall, about 39 percent of our students received a grade of B- or higher and about 66 percent received a C- or
higher. Among all the models, the corequisite group outperformed the other groups with 49 percent of students receiving a B– or higher and about 80 percent of students receiving a C– or higher.

The data representing the number of students who received a D+ or below and a C– or above in different course models were analyzed using the chi-square test (Table 2) to determine the association between the course model and grades (pass/fail). As mentioned in the results, we found that the course model and these grades are dependent.

The ANOVA test (Table 3) showed that students’ grades in the QR course were significantly higher when they completed the course under the corequisite model compared to the prerequisite model. There was no statistically significant difference between the corequisite model and QR alone.

Our results indicate that under the corequisite model, where students received the required support while enrolled in the QR course, the remedial education became more effective compared to the prerequisite model. The supplemental 90-minute session allowed the instructor and students to create a more active learning environment in which students were able to better integrate the remedial material with the QR course. From our own experience with the corequisite model, the instructor had greater flexibility in utilizing the supplemental session and was able to address the challenges faced by students in the QR course. Additionally, we felt students were less rushed and more relaxed with course materials in the corequisite model.

Among the many models that have been practiced in remedial education, the corequisite model has a unique advantage as it can be customized to the needs of individual institutions and their student body. It provides immense flexibility for the instructor to design and implement this model based on the needs of students who are enrolled in the credit-bearing course. We observed an added benefit to this model when the instructor who teaches the credit-bearing course also instructs the additional time devoted for remedial education. The instructor’s familiarity and understanding of the students in the credit-bearing course enabled him or her to identify the essential skills that students are lacking and that awareness could be used to shape the remedial education students receive under the corequisite model.

One important benefit of the pedagogical approach in the QR course (under corequisite model) was its ability to accelerate the learning process and completion of this course. Students in the prerequisite model, however, could not benefit from simultaneous integration of remedial topics with QR content. The student survey (Table 5) reflected that they perceived the prerequisite model as an apparent setback since they were doing only a 1-credit remedial course the entire semester. Many students expressed their disappointment towards this approach and, as a result, they did not exhibit the level of confidence and motivation required in an academic environment. In addition to this, students tended to forget the skills attained through the remedial work when they took the QR course in the next semester. These students also found it less interesting to engage in arithmetic and algebraic exercises that they had attempted several times during their school years. The corequisite model eliminated these factors. In fact, in the corequisite model the required skills were applied simultaneously to solve the real-world problems. This approach provided students an opportunity to appreciate the relevance of mathematics in their daily lives as well.

**Question 1:** What do you consider to be the strengths of this course?

**Student responses:**

1. It teaches you how to apply problem solving in everyday life.
2. I think the material taught in this course is very important.
3. It helps build a solid math base that you can expand from throughout college.
4. I have never taken a class like QR before, let alone heard about it and I must say I think this is a great critical thinking class. It helped me learn to look at math in different ways towards what is going on in life and how I can apply it to my life.
5. The strength of this course is that they don’t teach you useless math. It might be a remedial class; however they teach you a lot of important things-stuff that you can use every day like dosage requirements, which will be required in majors like Nursing. They also teach how taxes and mortgages are calculated with everyday class.

**Question 2:** If there are any other comments you would like to make about your perception of this course and/or the teaching of it, please do so here.

**Student responses:**

1. Please avoid having students take prerequisite, as the time they use for it should be spent on QR course to better understand the course and practice.
2. Reconsider the idea of students taking two math classes if students need more help with math then using the hour period of prerequisite course to go over homework or taking quizzes can definitely help improve math skills in QR course.
3. The labs (additional 90 minutes) were just what students need, more practice because math is all about hands on and doing out problems. The instructor was there for any questions we had and went over anything we needed clarification on. Being in the lab really helped me and I honestly think for a class like that lab should be mandatory.

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Table 5:
Although the corequisite and prerequisite models spent the same total class time with an instructor, the key distinction is that within the corequisite model students received the support when they needed it rather than in the previous semester as in the prerequisite model. Additionally, the students in the corequisite model were able to connect between the remedial hour instruction and their QR learning materials in a more consistent manner than the students in the prerequisite model. Both models were designed to encourage student-faculty interaction, however, it was more frequent and rewarding to the students in the corequisite model. They never felt completely lost due to the timely access to the instructor and the subsequent support mechanism in place. This approach has generated a higher level of motivation along with confidence and satisfaction (displayed in student survey) among the students under the corequisite model.

We used an anonymous survey at the end of the semester to better understand student perceptions and attitudes toward both the corequisite and prerequisite models. This evaluation was administered as an online survey with 10 questions containing both open-ended and closed-end questions. A total of 77 students completed the survey (35 of those from QR corequisite model and 42 from prerequisite model). In Table 5, on the previous page, we provide some of the sample (open-ended) questions and responses from the students who completed the survey. These responses strengthen the rationale for curriculum changes that we have undertaken in replacing the algebra courses with the QR course. We received many similar comments that reflected the overall enthusiasm students expressed towards the corequisite model. Additionally, the survey revealed that about 80 percent of the student body favored corequisite model as opposed to just 30 percent that responded positively to the prerequisite model.

Studies have well established that each lesson in a mathematics classroom should take into account students’ motivation level and dispositions and have as a goal the development of these affective characteristics (Brahier, 2011). Student success in the QR course, similar to any other courses, would also depend on non-cognitive skills such as attitude toward learning, motivation, autonomy, willingness to seek and accept help, desire to affiliate with peers or instructors, or willingness to expend effort on academic tasks (Anthony, 2000; Sedlacek, 2004). In short, teaching QR requires a much broader effort than simply helping students acquire skills and problem-solving strategies and the corequisite model of instruction can be a valuable tool in achieving that goal.

Conclusion and Future Work

One of the important aspects of this study was the positive effect of the corequisite model on student learning and their perceptions as described in the above section. We want to emphasize that although the learning outcomes were identical between the two models, the students under the corequisite model not only performed better than the prerequisite group, but had expressed a higher level of enthusiasm toward the learning process along with active engagement in classroom. We acknowledge that a single study from an institution covering one academic year is not sufficient to authenticate the validity of the corequisite model. However, our findings are supportive of the growing relevance of the corequisite model in remedial education.

The body of evidence is growing that the corequisite model can significantly improve the student success and has many advantages over other remedial mechanisms. Based on our study, beginning Fall 2015, Regis College has adopted the corequisite model as the sole remediation mechanism for all underprepared students in QR and eliminated the prerequisite model all together. It is worthy to note here that many other institutions of higher education are also following the corequisite model as their primary mode of remediation (Transforming Remediation in Georgia, 2015; Smith 2015). The benefit of the corequisite remediation could be extended to other introductory level college courses such as college algebra or precalculus. Institutions seeking to improve the remedial education in STEM fields can gain valuable experience by experimenting with this model. The corequisite model may vary depending on discipline, institution, course and even instructor. They all have a common focus not just on the goal of improving remedial course completion but more significantly, according to Complete College America, on completion of credit-bearing college courses that put students on a steadier path to completion of their degree (Complete College America, 2012).

The corequisite model may not serve the needs of all students especially those students in high need of remediation, but it can serve the needs of a large proportion of underprepared students. As such, continued research is needed to figure out for what group of students with remediation requirement the corequisite model will be most appropriate. This model provides many opportunities for students and instructors alike in order to create a learning environment that is more effective than the traditional remedial approaches.

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